Optimal Income Tax Policy and Wage Subsidy

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Abstract:
We show that in an imperfectly competitive economy, if the government cannot use wage subsidy, in a steady state and in the initial period the optimal labour income tax rate is zero. In an imperfectly competitive economy, since investment is primarily triggered by the motive to earn higher profits, over accumulation of capital induces suboptimal level of working hours. We argue that if the government is restricted to subsidize wage, the optimal policy should set zero tax on labour income which will encourage workers to increase working hours back to the optimal level.

Keywords: Optimal taxation, Monopoly power, Ramsey policy.
JEL Codes: D42, E62, H21, H30.

Introduction.

In this paper, we show that in an imperfectly competitive economy, if the government is not permitted to use wage subsidy, the optimal labour income tax rate in the initial period and in a steady state is zero. This policy weakens the government’s motivation to tax capital. In an imperfectly competitive economy, market power pushes private returns to labour and capital at a lower level than the socially optimal returns. In order to achieve the socially optimal outcome, the best policy should use a lump sum tax and should subsidize the monopoly distorted returns which in turns pushes private returns up to socially optimal returns. We consider two restrictions on this first best outcome.

First, we assume that lump sum taxes are not available, which motivates the standard Ramsey problem of choosing implementable allocations in order to maximize welfare. Secondly, we consider a restriction on using wage subsidies. Wage subsidies are typically associated with disincentive to work, since they provide strong incentives to misreport working hours in order to collect the subsidy. In the Ramsey problem, we add a constraint that restricts the government from using wage subsidy in imperfectly competitive sector. We find that with these two restrictions, in a steady state and in the initial period, optimal labour income tax rate is zero. Thus in an imperfectly competitive economy, if it is not possible to subsidize wage, the optimal policy is not to tax it.

It is well known that in an imperfectly competitive economy, profit seeking investment results in an over accumulation of capital which increases the marginal product of labour. But since private return to labour is monopoly distorted, working hours is suboptimal. We argue that a zero labour income tax policy is optimal since it encourages work and assists in bringing back the working hours to the socially optimal level.

Our study also contributes by extending the analysis by Judd (1997), Judd (2002) and Guo and Lansing (1999) who emphasize optimal capital income taxation in an imperfectly competitive economy. We show that in such an economy, not allowing the government to use wage subsidies weakens the government’s motivation to tax capital. This in turns shows that in an imperfectly competitive economy, the government’s motivation to subsidize capital is in fact insensitive to any changes in tax code, but if one restricts the government from using wage subsidies, the change in welfare effect of investment affects welfare and magnitude of the optimal capital tax/subsidy rate.
The Decentralized Economy.

Time is discrete and runs forever. The economy has two production sectors: sector \( y \) (the competitive sector) produces final goods (numeraire), and sector \( z \) (monopoly sector) produces a continuum of intermediate goods. The two technologies are:

\[
y_j = \left( \int_0^1 \frac{1}{\sigma} \left( \frac{1}{z^{1-\sigma}} \right) dz \right)^{1-\nu} n_{jt}^{1-\nu}, \quad \nu \in (0,1); \sigma \in (0,1) \quad (1.1)
\]

\[
z_j = k_m a n_{zt}^{1-\alpha}, \quad \alpha \in (0,1) \quad (1.2)
\]

where \( n \) is working time, \( k \) is capital and \( j \) is the level of \( j \in [0,1] \). Firms in monopoly sector exploit demand for intermediate goods and make positive economic profits \( \pi \). In a symmetric equilibrium, \( t \left( s, g \right) = \pi = \pi \left( s, g \right) \), \( t \left( s, g \right) = \pi = \pi \left( s, g \right) \)

\[
c_i + g_i + k_{i+1} = k_i a n_{zt}^{1-\alpha} n_{jt}^{1-\nu} + (1-\delta)k_i; \quad \delta \in (0,1) \quad (2)
\]

where \( c_i \) is private consumption, and \( g_i = \bar{g} > 0 \) is government’s consumption expenditure. The government’s budget constraint (with symmetry) is:

\[
\tau_c c_i + \tau_y w_{yt} n_{yt} + \tau_z w_{zt} n_{zt} + \theta (r_i k_i + \kappa \pi_i) + R_i^{-1} b_{i+1} - b_i - g_i = 0 \quad (3)
\]

where \( R_i \) is rate of return on real government bonds \( b \), and the tax rates are \( \tau_c \), \( \kappa \tau_y \), \( \tau_z \) and \( \tau_s \), \( s \in \{ y, z \} \), for capital, profits, private consumption and labor, respectively.\(^1\) The representative household’s problem is:

\[
\max_{\{c_i, n_{yt}, n_{zt}\}} \sum_{t=0}^{\infty} \beta^t u\left(c_i, 1-n_{yt} - n_{zt}\right)
\]

s.t. \((1 + \tau_c) c_i + k_{i+1} + R_i^{-1} b_{i+1} = (1-\tau_y) w_{yt} n_{yt} + (1-\tau_z) w_{zt} n_{zt} + [(1-\tau_y) r_i + (1-\delta) \kappa] k_i + b_i + (1-\kappa \tau_y) \pi_i \)

\[
k_0 > 0, b_0 \text{ given} \quad (4)
\]

with \( \beta \in (0,1) \). \( u(\cdot) \) satisfies standard regularity assumptions. The symmetric equilibrium conditions involve (2), (3), transversality conditions, prices and profit, and

\[
-u_{s t}(t) = u_{c}(t) \xi_t (1-\tau_y) w_{st} \quad \text{for } s = y, z
\]

\[
\beta [(1-\tau_{k+1}) r_{i+1} + 1-\delta] = R_i = \frac{u_{c}(t) \xi_{i+1}}{u_{c}(t+1) \xi_t}
\]

where \( \xi_t \equiv (1 + \tau_c)^{-1} \).

With \( \tilde{r} \equiv (1-\tau_y) r \) and \( \tilde{w}_z \equiv (1-\tau_z) w_z \), the equilibrium cost of capital and labour in monopoly sector are given by \( \tilde{r} = (1-\sigma)(1-\tau_k) MP_k \) and \( \tilde{w}_z = (1-\sigma)(1-\tau_z) MP_{wz} \), where \( MP_x \) is the

---

\(^1\) \( \kappa \in [0,1] \) implies the government’s set of tax treatments \{\text{not tax}, \text{at par with capital tax}\} for distributed corporate profits.
marginal physical product of factor $x$. Since the parameter $\sigma$ acts like a privately imposed tax, the best policy should subsidize capital income and labour income in monopoly sector. If the objective of this subsidy is to push up the private return equal to socially optimal return, the best policy should include $\tau_k = \tau_z = \frac{-\sigma}{1-\sigma}$. With consumption taxation the best policy is not that simple. In fact, with consumption taxation, the best policy overcompensates wage income in monopoly sector.

To see this more clearly, consider the social planner’s problem, i.e. the problem where the planner chooses allocations to maximize welfare subject to resource constraint (2). The resulting first order conditions are the conditions consistent with the first best outcome. If one compares the conditions with the decentralized equilibrium for the current model, the policy that generates first best outcome includes a large lump sum tax, $\tau_{zt} = \tau_r, \tau_{yt} = -(\sigma + \tau)(1-\sigma)^{-1}, \text{ and } \tau_{kt} = -\sigma(1-\sigma)^{-1}$. The first best capital income subsidy does exactly what it should do; it pushes the private return up to socially optimal return to capital. With the wage subsidy in monopoly sector, $w_z = (1+\tau)MP_z$, which simply means wage income from monopoly sector is overcompensated. In addition, the first best policy subsidizes wage income in competitive sector. This is at the expense of a large lump sum tax and a consumption tax.

We consider a case where lump sum taxes are not available. Since the government has a strong motivation to subsidize wages, we consider a case where in addition to no lump sum taxes, the government cannot use wage subsidy. For simplicity, we assume that the government faces a restriction on using subsidy in monopoly sector.

The Ramsey Problem.

We use primal approach to derive the conditions that characterize the Ramsey allocation. Then we look for the taxes that can implement the second-best wedges. The Ramsey allocation can be characterized by choosing an allocation. The government chooses the allocation \(\{c_t, n_{yt}, n_{zt}, k_t, h_t\}_{t=0}^\infty\) so as to maximize welfare subject to the resource constraint (2), and an implementability constraint that ensures that resulting taxes, allocations and prices are consistent with decentralized equilibrium. In order to derive the implementability constraint, we iterate the household’s budget constraint backwards and derive its present value version. The present value constraint is:

\[
\sum_{t=0}^{\infty} q_t^g \left[ (1+\tau_{ct})c_t - (1-\tau_{yt})w_{yt}n_{yt} - (1-\tau_{zt})w_{zt}n_{zt} - (1-\tau_{kt})k_t \right] = \left[ (1-\tau_{kt})r_0 + 1 - \delta \right]k_0 + b_0
\]

where $q_t^g = \left( \prod_{s=1}^{t} R_s \right)^{-1}$ with $q_0^g = 1$.

Notice that the disincentive effect of wage subsidy is stronger if the government has access to consumption taxes\(^2\). Generally, government spending is large relative to the difference between consumption and wage income. Expressed as a share of GDP, the average difference between consumption and wage income in the US is approximately 0.14, while average government expenditure’s share in GDP is approximately 0.23\(^3\). From (6.1), the present value of private and government consumption must equal the present value of wage and profit income plus the value of

\(^2\) Coleman II (2000) constructs an optimal taxation model with consumption and income taxes assuming that all markets are perfectly competitive, and argues, from his calibration, that the US economy could attain maximum welfare gains from switching to Ramsey policy if the government is prohibited to use wage subsidy.

\(^3\) This is based on the 1947-2005 quarterly series of Gross Domestic Product (GDP), Personal Consumption Expenditure (PCEC), Government Expenditures (GCE) and Wages and Salary Accruals (WASCUR), from Federal Reserve Bank of St. Louis, Economic Data-FREDII.
initial assets. Since the consumption-income difference is generally small relative to the government expenditure, a combination of wage subsidy and a consumption tax induces a very large tax/subsidy rate. Large wage subsidy is infeasible since it provides incentives to misrepresent hours worked. In addition, large consumption tax rates may well lead to unreported barters.

With (6.1), we rerun the representative household’s optimization problem. Using the set of decentralized equilibrium conditions, we substitute out prices and taxes in the present value budget constraint in order to derive an intertemporal constraint that involves initial conditions and allocations, which is:

\[
\sum_{t=0}^{\infty} \beta^t [u_c(t) c_t + u_{nc}(t) n_{y,t} + u_{ac}(t) n_{z,t} - u_c(t) \bar{\zeta}_t (1 - \kappa \tau_{zt}) \pi_t] - \tilde{\Omega}(c_0, n_{y,0}, n_{z,0}, \tau_{k,0}, \xi_0) = 0
\]

where

\[
(1 - \kappa \tau_{zt}) \pi_t = \left\{ \begin{array}{ll}
\left[ v \sigma (1 - \kappa) k_{it}^{av} n_{zt}^{(1-\alpha) - 1} n_{yt}^{1-\nu} + k_{it} \kappa \sigma \alpha (1 - \sigma) \right]^{1-\delta} u_c(t-1) \xi_{t-1}^{1-\delta} / \beta u_c(t) \xi_t^{1-\delta} - (1 - \delta) \\
(1 - \kappa \tau_{zt}) (v \sigma) k_0^{av} n_{z0}^{(1-\alpha) - 1} n_{y0}^{1-\nu} & \text{for } t = 0
\end{array} \right.
\]

and, \( \tilde{\Omega}(c_0, n_{y,0}, n_{z,0}, \xi_0, \tau_{z0}) \equiv u_c(0) \xi_0 \{ (1 - \tau_{z0}) \alpha (1 - \sigma) \nu (k_0)^{-1} (y_0 + (1 - \delta)) k_0 + b_0 \} . \) The constraint characterizing non-negative wage taxation in monopoly sector is:

\[
\xi_t(1 - \alpha) \nu (1 - \sigma) k_{it}^{av} n_{zt}^{(1-\alpha) - 1} n_{yt}^{1-\nu} + u_{nc}(t) u_c(t) [u_c(t)]^{-1} \geq 0
\]

The Ramsey problem is the government’s problem of choosing allocations and \( \{ \xi_t \}_{t=0}^{\infty} \) to maximize welfare subject to constraints (2), and (6.2). Let \( \overline{\Phi} \geq 0, \{ \overline{\xi}_t \}_{t=0}^{\infty} \) and \( \{ \overline{\mu}_t \}_{t=0}^{\infty} \) represent the multipliers for (6.2a&b), (2), and (6.2c), respectively, and denote the pseudo utility function as \( \overline{V}(c_t, n_{y,t}, n_{z,t}, k_t, \xi_t, \Phi) \). The pseudo utility function takes the form:

\[
\overline{V}(c_t, n_{y,t}, n_{z,t}, k_t, \xi_t, \Phi) = u_c(1 - n_{y,t} - n_{z,t}) + \Phi [u_c(t) c_t + u_{ac}(t) n_{y,t} + u_{ac}(t) n_{z,t} - u_c(t) \bar{\zeta}_t (1 - \kappa \tau_{zt}) \pi_t]
\]

with (6.2b). The first order conditions for \( t \geq 1 \) are equations (6.3a-e), as follows:

\[
\begin{align*}
\overline{V}_{c_t} & - \overline{\xi}_t - \overline{\mu}_t \left[ u_{ac}(t) u_c(t) - u_c(t) u_{ac}(t) / u_c(t) \right] = 0 \\
\overline{V}_{n_{y,t}} + (1 - \nu) k_{it}^{av} n_{zt}^{(1-\alpha) - 1} n_{yt}^{1-\nu} [\overline{\xi}_t + \overline{\mu}_t \nu (1 - \alpha) (1 - \sigma) n_{zt}^{-1}] + \overline{\mu}_t \left[ u_c(t) u_{ac}(t) - u_{ac}(t) u_{ac}(t) / u_c(t) \right] = 0 \\
\overline{V}_{n_{z,t}} + \nu (1 - \alpha) k_{it}^{av} n_{zt}^{(1-\alpha) - 1} n_{zt}^{-1} [\overline{\xi}_t + \overline{\mu}_t \nu (1 - \alpha) (1 - \sigma) [\nu (1 - \alpha) - 1] n_{zt}^{-1}] + \overline{\mu}_t \left[ u_c(t) u_{ac}(t) - u_{ac}(t) u_{ac}(t) / u_c(t) \right] = 0 \\
\overline{V}_{\xi_t} - \beta \left[ \overline{V}_{c(t+1)} + \overline{\xi}_t \nu [\alpha k_{zt}^{(1-\alpha) - 1} n_{zt}^{1-\nu} + (1 - \delta)] + \overline{\mu}_t \nu (1 - \alpha) (1 - \sigma) \nu (1 - \alpha) n_{zt}^{1-\nu} / n_{zt+1} \right] = 0 \\
\overline{V}_{\xi_t} + \overline{\mu}_t \nu (1 - \alpha) (1 - \sigma) k_{it}^{av} n_{zt}^{(1-\alpha) - 1} n_{zt}^{-1} = 0
\end{align*}
\]

The Kuhn-Tucker condition is:

\[
\xi_t(1 - \alpha) \nu (1 - \sigma) k_{it}^{av} n_{zt}^{(1-\alpha) - 1} n_{yt}^{1-\nu} + u_{nc}(t) u_c(t) [u_c(t)]^{-1} \geq 0, \text{ with equality if } \overline{\mu}_t > 0
\]

(6.3f)
The derivative of the Pseudo utility function with respect to $t$ is:

$$
\tilde{\nu}_t(t) = -\tilde{\Phi}
\begin{bmatrix}
(1 - \kappa \theta_t) \pi_t u_c(t) + \tilde{\xi}_t u_c(t) \\
\frac{\kappa \sigma}{\alpha (1 - \sigma) \beta}
\begin{bmatrix}
k_{t-1} u_c(t-2) - k_t u_c(t-1) \tilde{\xi}_{t-1}
\end{bmatrix}
\end{bmatrix}
$$

(6.4a)

where $(1 - \kappa \tau_c) \pi_t$ is given by (6.2b). Consider the steady state and the initial period. Substitute (6.4a) in (6.3e) and evaluate the expression in steady state. This results in:

$$
\tilde{\Phi}(1 - \kappa \tau_c) \pi_0 u_c(0) + \tilde{\mu}_{z_0} (1 - \alpha) \nu (1 - \sigma) k^{av} n_z^{(1 - \alpha)^{-1}} n_{y_0}^{1 - \nu} - \tilde{\Phi} R_0 k_0 u_c(0) = 0
$$

(6.4b)

**Proposition 1:** If the government is not permitted to subsidize wage in monopoly sector, in period 0 and in a steady state the optimal wage tax rate in the monopoly sector is zero.

**Proof:** Equation (6.4b) implies that $\tilde{\mu}_{z} > 0$, which in turns imply that in the steady state the non-negativity constraint (6.2c) is satisfied with equality, and $\tau_z = 0$. The first order condition to the Ramsey problem with respect to $\tilde{\xi}_0$ is:

$$
- \tilde{\Phi}(1 - \kappa \tau_z) \pi_0 u_c(0) + \tilde{\mu}_{z_0} (1 - \alpha) \nu (1 - \sigma) k^{av} n_z^{(1 - \alpha)^{-1}} n_{y_0}^{1 - \nu} - \tilde{\Phi} R_0 k_0 u_c(0) = 0
$$

(6.4c)

Equation (6.4c) implies $\tilde{\mu}_{z_0} > 0$, i.e. at $t = 0$ the non-negativity constraint (6.2c) is satisfied with equality, and $\tau_{z_0} = 0$.

The transition to this steady state is however not necessarily characterized by $\tilde{\mu}_{z_t} > 0$ for all $t \geq 1$. Although $\tilde{\mu}_{z_0} > 0$ and $\tilde{\mu}_z > 0$, starting at $t = 1$ many paths of tax rates can achieve $\tau_z = 0$ and all these paths may be consistent with the equilibrium behaviour of taxpayers at the optimal allocations. Some of these paths may have $\tau_{z_t} > 0$ along the transition because of the term

$$
\begin{bmatrix}
k_{t-1} u_c(t-2) - k_t u_c(t-1) \tilde{\xi}_{t-1}
\end{bmatrix}
\begin{bmatrix}
k_{t-1} u_c(t-1) \tilde{\xi}_{t-1}
\end{bmatrix}
$$

in (6.4a).

Evaluate (6.3d) in steady state and combine steady state version of (6.3a) and (5.1b) to derive:

$$
\tau_k = -\frac{\sigma}{1 - \sigma} - \left[ \frac{\tilde{\nu}_k + \tilde{\mu}_{z} \nu (1 - \alpha) n_z^{(1 - \alpha)^{-1}}}{\tilde{\nu}_c - \tilde{\mu}_{z} (u_{cc} u_c - u_c u_{cc}) (u_c)^{-2}} \right] (\nu)^{-1}
$$

(6.5)

(6.5) implies that in a steady state, the sign of the optimal capital tax rate is ambiguous, a result is which is perfectly consistent with that of Guo and Lansing (1999). In a steady state, the government has a motivation of subsidize capital income because of the term $\frac{\sigma}{1 - \sigma}$. This is exactly equal to the first best capital subsidy rate. In addition, due to profit seeking investment that distorts welfare, the government has a motivation to tax capital, which is represented by the second term. If one considers the Ramsey problem where the government faces no restriction on using wage subsidy, similar steps derives:
Since the Pseudo utility function is defined as welfare plus the social cost of distorting taxes times the change in welfare due to distorting taxes, it represent a measure of second best welfare. The derivatives $V_k$ and $V_c$ represent the marginal effect of capital accumulation and consumption on second best welfare. Their ratio in (6.6), therefore, is a measure of the relative effect of investment in physical capital on second best welfare. Unlike a competitive markets setting, profits appear in the implementability constraint, implying that investment induces a direct effect on second best welfare. This effect is not perceived by households. With or without the wage subsidy restriction, it is straightforward to show that this ratio is negative.

From (6.5) and (6.6), relative to the scenario with no wage subsidy restriction, the government’s motivation to tax capital is weaker in the Ramsey policy with a wage subsidy restriction. Thus if the government cannot subsidize wage in monopoly sector, in a steady state its motivation to tax capital in monopoly sector is weaker. Intuitively, this fiscal setting creates feasible platform for the government to subsidize capital income.

**Concluding Remarks.**

This paper shows that in an imperfectly competitive economy if the government cannot subsidize wage, the optimal tax policy involves zero wage tax and a weaker motivation to tax capital income. In an imperfectly competitive economy, both labour and capital suffer a loss of return due to monopoly distortion. Investment in such economies is primarily triggered by a motivation to earn higher profits. Such over accumulation of capital induces significant loss in welfare, but more capital per worker increases the productivity of labour. This results in a higher (but sub optimal) real wage in a monopoly-distorted sector and a suboptimal level of working hours. Any additional capital provides incentives for workers to reduce the working hour further low. Any additional capital also makes the government’s motivation to tax capital stronger. Distortion neutralizing may therefore require subsidizing wage in monopoly sector in order to encourage work, and tax capital to discourage over accumulation of capital. Since wage subsidies can overcompensate labour income, and can be counter effective in reducing working hours, we consider the Ramsey problem with a restriction on wage subsidy. We find that with such restriction, the government’s optimal choice is to set wage tax equal to zero in the long run and in the initial period.

**References.**


